Stat 20: Discussion at section #1

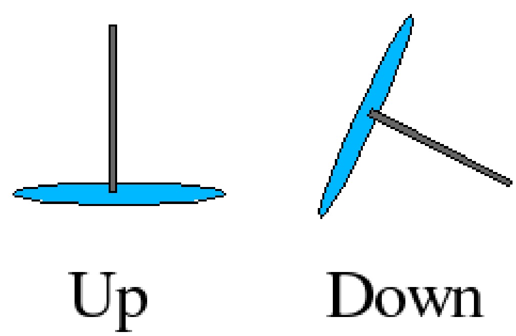
Ben Bolstad: bolstad@stat.berkeley.edu

Aug 27, 2003

1 Problem 1

Drop a thumb tack 100 times. Count how many times it lands in the "up" position. Now estimate the P(tack lands "up").

What might effect this probability? If we tossed different types of thumb tack would we get different estimates?



Solution: After tossing the tack 100 times we will have a count of the number of heads x. The estimate of the probability of getting an "up" is given by P(tack lands "up") = x/100.

Many things might effect the probability. For instance: the height and position you hold the tack before dropping it, the surface you are dropping it onto. A different type of tack would likely have a different probability. In class we used a tack with a flat base, with a round base we would perhaps have a different outcome.

2 Problem 2

Lets play a game. Suppose we toss a coin 4 times. You win \$1 for each head and lose \$1 for each tail.What are the possible outcomes (what are are possible winnings)?

Now toss a coin four times and record your winnings (or loss). Repeat a number of times to estimate the probability of each of the outcomes.

What is the theoretical probability of each of the outcomes (assume that we have a fair coin)? Suppose that the coin is not fair, instead assume that P(Heads) = 0.25 and P(Tails) = 0.75. What is the probability of each of the outcomes now?

Solution: The possible winnings are -\$4,-\$2, \$0, \$2 and \$4. Since three heads would put us \$3 ahead and one tails would subtract \$1 giving us \$2. Similarly two heads and two tails cancel out for \$0 and three tails and one head would give us -\$2.

Repeating the coin toss game a number of times we can estimate the probability of each of the outcomes. We would estimate P(win -\$4) = number of times we end up with -\$4/ number of times we play the game and so on ....

To compute the theoretical probability of each outcome, look at all possible outcomes (each being equally likely since we are dealing with a fair coin).

Outcome	Winnings
HHHH	\$4
HHHT	\$2
HHTH	\$2
HHTT	\$0
HTHH	\$2
HTHT	\$0
HTTH	\$0
HTTT	-\$2
TTTT	-\$4
TTTH	-\$2
TTHT	-\$2
TTHH	\$0
THTT	-\$2
THTH	\$0
THHT	\$0
THHH	\$2

There are 16 different equally likely outcomes. There is one way of winning \$4 so P(Win \$4) = 1/16, four ways of winning \$2 so P(Win \$2) = 1/4, six ways of ending with \$0 so P(Win \$0) = 6/16, four ways of getting -\$2 so P(Win -\$2) = 1/4, and one way of winning -\$4 so P(Win -\$4) = 1/16.

If we now assume that the coin is not fair the we must compute the probability of each outcome. Since coin tosses are independent, we compute the probability by multiplying the probability of each of the individual events. So P(THHT) = P(T)P(H)P(H)P(T) = 0.75*0.25*0.25*0.75 = 0.03515625 etc. The following table gives the probability of each of these events.

Outcome	Probability	Winnings
HHHH	0.00390625	$4
HHHT	0.01171875	$2
HHTH	0.01171875	$2
HHTT	0.03515625	$0
HTHH	0.01171875	$2
HTHT	0.03515625	$0
HTTH	0.03515625	$0
HTTT	0.1054688	-$2
TTTT	0.3164062	-$4
TTTH	0.1054688	-$2
TTHT	0.1054688	-$2
TTHH	0.03515625	$0
THTT	0.1054688	-$2
THTH	0.03515625	$0
THHT	0.03515625	$0
THHH	0.01171875	$2

In this case P(Win $2) = 4* 0.01171875 = 0.046875 and similarly for each of the other outcomes.

In a later class we will discuss a probability distribution we can use to solve this sort of problem.

3 Problem 3

Hypothetical probabilities for a person in the USA being in each of the possible blood groups is given in the following table.

Blood Type	A	B	AB	O
Probability	.40	.11	?	.45

What is the probability of picking a person at random from the USA is of bloodtype AB? Blood type O is the universal donor type (it can be safely given to anyone of any other blood type). Suppose someone of blood type B needs a transfusion, what is the probability of picking a person at random from the population who can provide a compatible blood type?

Solution: There are only 4 possible blood types and a person can have only one blood type (they are mutually exclusive). Therefore the sum of the probabilities of each of the blood types should add to 1. Therefore P(AB) = 1 -(P(A) + P(B) + P(O)) = 1 - .4 - .11 - .45 = .04. Since blood type B and blood type O can safely be given to a blood type B receipient P(Suitable Donor) = P(B $\cup$ O) = P(B) + P(O) = .45 + .11 = 0.56.

4 Problem 4

Suppose that the probability for a person living in New Zealand being of each of the possible blood types is given in the following table.

Blood Type	A	B	AB	O
Probability	.27	.26	.12	.35

We will pick two people at random. 1 from the USA and 1 from New Zealand. What is the probability that they are both 0? What is the probability they are both of the same blood type? What is the probability of picking 1 person who is A and 1 person who is B?

Solution: Since the outcome of picking a person from New Zealand is independent from the outcome of picking a person from the USA P(O from USA ∩ O from NZ) = P(O from USA)P(O from NZ) = .45(0.35) = 0.1575.

If we pick two people of the same blood type there are four possible ways this can happen, both A, both B, both AB or both O. So

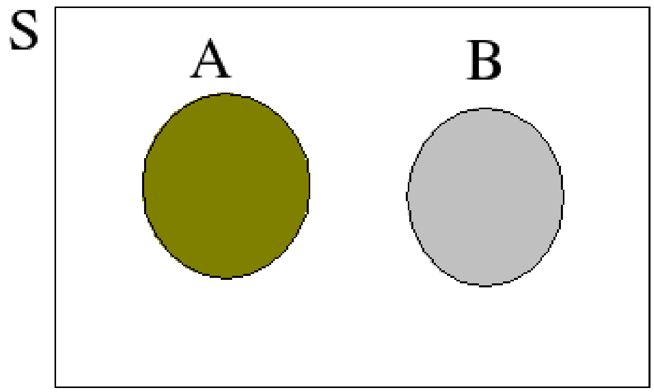
P(Both of same blood type) = P(A from USA ∩ A from NZ) + P(B from USA ∩ B from NZ) + P(AB from USA ∩ AB from NZ) + P(O from USA ∩ O from NZ)
= P(A from USA)P(A from NZ) + P(B from USA)P(B from NZ) + P(AB from USA)P(AB from NZ) + P(O from USA)P(O from NZ)
= .40(.27) + .11(.26) + .04(.12) + .45(.35)
= 0.2989

There are two possible ways that we can get a person of type A and one of B. Either A from USA/B from NZ or B from USA and A from NZ. Therefore

P(1 person is A and 1 person is B) = P(A from USA ∩ B from NZ) + P(B from USA ∩ A from NZ)
= P(A from USA)P(B from NZ) + P(B from USA)P(A from NZ)
= .40(.26) + .11(.27) = .1337

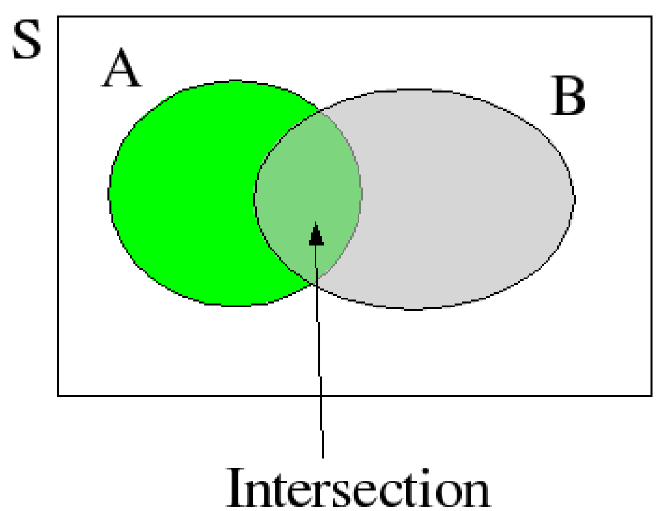
5 Miscellanous Material

We define two events as being **mutually exclusive** if they are unable to be both true at the same time. For example suppose we roll two dice. Let event A = sum is nine and let B = roll a double. Then A and B are mutually exclusive, since it is not possible for both events to occur simultaneously (try writing down all the possibilities to see that this is true). We can use a Venn diagram to illustrate this possibility. Note that the events A and B do not overlap.



We define two events as being **independent** if the result of the second event is not affected by the result of the first event. For example suppose we have a bucket containing three red balls and four white balls. Then we make two drawings from the bucket replacing the balls after each drawing. The probability of drawing a red ball on the second draw is independent of what happened on the first draw. Now suppose we don't replace the ball after the first draw then the probability of drawing a red ball on the second draw is **dependent** on the first draw.

You can think of **union** is equivalent to "or" and **intersection** as equivalent to "and". An event is in the Union of A and B if it occurs in both A or B or both A and B. An event if in the intersection of A and B if it is in both A and B. The following venn diagrams demonstrate union and intersection. The intersection is the region of events in both A and B. The union is all events in shaded area (not matter whether this is grey, green or grey/green).



If two events A and B are independent then the probability of both events occuring is the product of the probabilities ie

$P(A \cap B) = P(A)P(B)$

If A and B are dependent and A occurs before B then

$P(A \cap B) = P(A)P(B \mid A)$

where $P(B \mid A)$ means the probability of B happening given A happened.

If A and B are mutually exclusive then

$P(A \cup B) = P(A) + P(B)$

If A and B are not mutually exclusive then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$