Stat 20: Discussion at section

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Hypothesis tests

To perform a hypothesis test we need to do the following things.

(a.) State a null and alternative hypothesis

(b.) Compute a test statistic

(c.) Look up the P-value (or find the acceptance/rejection regions)

(d.) Interpret the result

Some common null and alternative hypotheses

We use H_0 to represent the null and H_A to represent the alternative (sometimes people use H_1). Note that the following are for testing proportions (you would substitute μ for p to test means).

Two sided alternative

$$H_0 : p = p_0 \text{ vs } H_A : p \neq p_0$$

One sided alternatives

$$H_0 : p \leq p_0 \text{ vs } H_A : p > p_0$$

or

$$H_0 : p \geq p_0 \text{ vs } H_A : p < p_0$$

where p_0 is a given proportion.

What is the null hypothesis and what is the alternative?

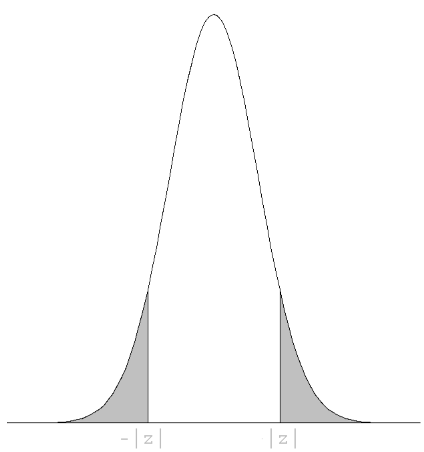
Usually the null is a statement of no effect or no change or no difference (hence it includes the equals) and the alternative is a possible change. Usually it is easier to set things up by first figuring out what the alternative hypothesis (this is usually what we want to find evidence for) then defining the null. For example if we are interested in whether there has been an increase in p then the alternative hypothesis is $H_A : p > p_0$ and the null hypothesis is the opposite ie $H_0 : p \leq p_0$

A test statistic for testing hypotheses about a single proportion

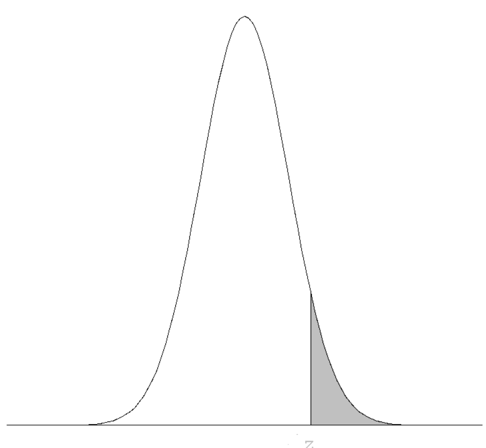
$$z = \frac{\hat{p} - p_0}{\sqrt{\hat{p}(1 - \hat{p})/n}}$$

P-values for these tests

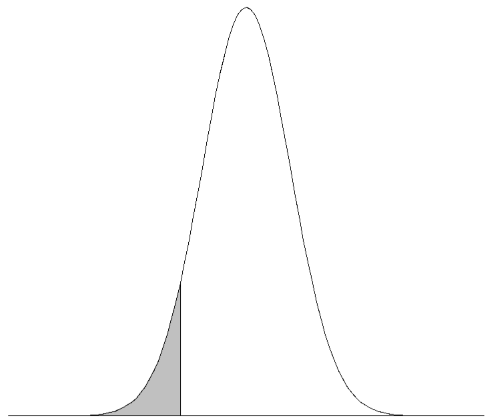
$H_0 : p = p_0$ vs $H_A : p \neq p_0$ P-value $2P\left(Z \geq |z|\right)$



$H_0 : p \leq p_0$ vs $H_A : p > p_0$ P-value $P\left(Z \geq z\right)$



$$H_0 : p \geq p_0 \text{ vs } H_A : p < p_0 \text{ P-value } P(Z \leq z)$$



where p_0 is a given proportion. The normal probability table is used to look up these probabilities.

What exactly is a P-value?

The P-value is the probability if we assume that H_0 is true, that the test statistic would take on a value as extreme or more extreme than the value we actually observed. A smaller P-value gives us stronger evidence against the null hypothesis.

Interpreting P-values

When carrying out a hypothesis test we usually use the P-value to decide whether or not the null hypothesis should be rejected. The smaller the P-value the more sure we feel about rejecting the null hypothesis. You can view the P-value as a measure of the strength of the evidence against the null hypothesis (and for the alternative). The smaller the P-value the stronger the evidence against the null. For example a P-Value of 0.1 would be considered (very) weak evidence, of 0.05 would be evidence, 0.01 strong evidence and .001 very strong evidence.

Sometimes a fixed level of significance is used. In this case we compare our P-value with the level of significance and reject the null in favor of the alternative if our P-value is smaller than the level of significance, otherwise we fail to reject the null. For example with a 5% level of significance we would reject the null only if our P-value was smaller than 0.05. The smaller the level of significance, the greater the evidence we require to reject the null hypothesis.

Note that a hypothesis test allows you to prove that the alternative is hypothesis is true, but you can not really prove that the null hypothesis is true (and therefore it is more proper to say that you cannot reject the null or fail to reject the null then to say that you accept the null).

Question 1

The English mathematician John Kerrich tossed a coin 10,000 times and obtained 5067 heads. Was his coin fair?

Answer: Let p be the probability of getting a heads on a toss of his coin. Our null and alternative hypotheses are

$$H_0 : p = 0.5 \text{ vs } H_A : p \neq 0.5$$

Our estimate pf p is $\hat{p} = \frac{5067}{10000} = 0.5067$ and so the test statistic is

$$z = \frac{0.5067 - 0.5}{\sqrt{0.5067(1 - .5067)/1000}} = 1.34$$

we are using the two sided alternative and so the P value is given by

$$2P\left(Z \geq 1.34\right) = 2P(Z < -1.34) = 2(0.0901) = 0.1802$$

since our P-value is large we can not reject the null hypothesis. In other words we have no evidence to say that the coin is not fair.

Question 2

A starting player for a major college basketball team made only 3.2% of her free throws last season. During the off season she worked on developing a softer shot in the hope of improving her free throw accuracy. In the first eight games this season she made 22 out of the 42 free throws she attempted. Let p be her probabibility of making a free throw this season. Did she improve her accuracy?

Answer: We are interested in whether she has improved are accuracy this suggests that our null and alternative hypotheses should be

$$H_0 : p \leq 0.362 \text{ vs } H_A : p > 0.362$$

note that the estimate of p is $\hat{p} = \frac{22}{42} = 0.524$ and so our test statistic is

$$z = \frac{0.524 - 0.362}{\sqrt{0.524(1 - .524)/42}} = 2.10$$

and our P-value is given by

$$P\left(Z \geq 2.10\right) = 1 - P(Z < 2.10) = 1 - .9821 = 0.0179$$

Since this P-value is so small we have good evidence to reject the null hypothesis in favor of the alternative hypothesis (if we were doing the test at the 5% level of significance we would reject the null). Therefore we conclude that she has improved here accuracy.

Question 3

Last season a farmer lost 20% of his crop to an insect pest. This season he uses a pesticide to reduce this problem. Right before harvest he chooses at random 60 of his fields (from the many on his farm) to survey. He finds the insect in 9 of the fields. Did the pesticide reduce the insect problem?

Answer: Let p be the proportion of his crop contaminated by insects. Since we are interested in whether the proportion of the crop that is contaminated has been reduced the null and alternative hypotheses for this test should be

$$H_0 : p \geq 0.2 \text{ vs } H_A : p < 0.2$$

our sample estimate is $\hat{p} = \frac{9}{60} = 0.15$. The test statistic is

$$z = \frac{0.15 - 0.2}{\sqrt{0.15(1 - .15)/60}} = -1.08$$

and the P-value is given by

$$P\left(Z < -1.08\right) = 0.1401$$

which is large and do we have not evidence to reject the null hypothesis. We have no evidence to indicate that the insect problem has been reduced.