# Stat 20: Discussion at section

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## Hypothesis tests for two proportions

### Some common null and alternative hypotheses

We use $H_0$ to represent the null and $H_A$ to represent the alternative (sometimes people use $H_1$). Note that the following are for testing proportions (you would substitute $\mu$ for $p$ to test means).

#### Two sided alternative

$$H_0 : p_1 = p_2 \text{ vs } H_A : p_1 \neq p_2$$

#### One sided alternatives

$$H_0 : p_1 \leq p_2 \text{ vs } H_A : p_1 > p_2$$

or

$$H_0 : p_1 \geq p_2 \text{ vs } H_A : p_1 < p_2$$

### A test statistic for testing hypotheses about two proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2}}$$

### P-values for these tests

$$H_0 : p_1 = p_2 \text{ vs } H_A : p_1 \neq p_2 \text{ P-value } 2P\left(Z \geq |z|\right)$$

$$H_0 : p_1 \leq p_2 \text{ vs } H_A : p_1 > p_2 \text{ P-value } P\left(Z \geq z\right)$$

$$H_0 : p_1 \geq p_2 \text{ vs } H_A : p_1 < p_2 \text{ P-value } P\left(Z \leq z\right)$$

## Question 1

A university financial aid office polled a SRS of undergraduate students to study their summer employment. Not all students were employed the previous summer. 728 of the 817 men were employed and 603 of the 752 women were employed. Is there evidence that the proportion of male students employed differed from the proportion of female students employed?

*Answer*:

Let $p_1$ be the proportion of all male undergraduates who were employed. Let $p_2$ be the proportion of all female undergraduates who were employed. The null and alternative hypotheses are

$$H_0 : p_1 = p_2 \text{ vs } H_A : p_1 \neq p_2$$

the sample proportion estimates are $\hat{p}_1 = \frac{728}{817}$ and $\hat{p}_2 = \frac{603}{752}$. The test statistic is

$$z = \frac{\frac{728}{817} - \frac{603}{752}}{\sqrt{\frac{728}{817}\left(1 - \frac{728}{817}\right)/817 + \frac{603}{752}\left(1 - \frac{603}{752}\right)/752}} = 4.91$$

And so the P-value is $2P(Z > 4.91) = 2P(Z < -4.91) < 2P(z < -3.49) = 2(0.0002) = .0004$. We would reject the null hypothesis in favor of the alternative. There is a difference in the proportion of students employed during the summer between genders.

## Question 2

A clinical trial was conducted to examine the effectiveness of daily doses of aspirin in the treatment of strokes. Patients were randomized into treatment and control groups. Neither the physician nor the patient knew whether they were receiving the aspirin or a placebo tablet. After six months of treatment, the attending physicians evaluated each patient's progress as favorable or unfavorable. Of the 78 patients in the aspirin group, 63 had favorable outcomes. The control group of 77 patients had 43 patients with favorable outcomes. Let $p_1$ be the proportion of patients in the treatment group (aspirin) who had favorable outcomes after six months. Let $p_2$ be the proportion of patients in the control group (placebo) who had favorable outcomes after six months. Did the aspirin increase the chances of the patient having a favorable outcome? Perform a hypothesis and interpret your result.

*Answer*:

First the appropriate hypothesis test is

$$H_0 : p_1 \leq p_2 \text{ vs } H_A : p_1 > p_2$$

our sample estimates are $\hat{p}_1 = \frac{63}{78}$ and $\hat{p}_2 = \frac{43}{77}$. The test statistic is

$$z = \frac{\frac{63}{78} - \frac{43}{77}}{\sqrt{\frac{63}{78}\left(1 - \frac{63}{78}\right)/78 + \frac{43}{77}\left(1 - \frac{43}{77}\right)/77}} = 3.45$$

ans so the P-value is $P(Z > 3.45) = 1 - P(Z < 3.45) = .0003$. We would reject the null hypothesis and conclude that aspirin helps patients recover from strokes.

# Question 3

A particular pesticide is commonly used to treat infestations of the German cockroach, Blattella germanica. A study is carried out to investigate the persistence of this pesticide on various types of surfaces. Researchers applied a 0.5% emulusion of the pesticide to glass and plasterboard. After 14 days, the placed 36 cockroaches on each surface. The number of dead cockroaches was recorded after two days. On glass 18 died, on the plasterboard 26 died. Is there a difference in mortatility between the two surfaces. Compute a 95% confidence interval for the difference in mortality between the two surfaces. Carry out an hypothesis at the 5% level of significance to test whether there is a difference between the two surfaces.

*Answer*:

Let $p_1$ be the proportion of cockroaches who die on the glass. Let $p_2$ be the proportion of cockroaches who die on the plasterboard. Our sample estimates are $\hat{p}_1 = \frac{18}{36}$ and $\hat{p}_2 = \frac{26}{36}$. The standard error of the difference $\hat{p}_1 - \hat{p}_2$ is given by

$$\text{SE}\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{18}{36}\left(1 - \frac{18}{36}\right)/36 + \frac{26}{36}\left(1 - \frac{26}{36}\right)/36} = 0.1119$$

and so the 95% confidence interval is given by

$$\left(\frac{18}{36} - \frac{26}{36}\right) \pm 1.96(0.1119)$$

ans so the interval is $(-0.442, -0.003)$. Note that 0 is not in this interval and so we would conclude from the interval that there is a difference between the surfaces.

Our hypothesis test will be

$$H_0 : p_1 - p_2 = 0 \text{ vs } H_A : p_1 - p_2 \neq 0$$

and the test statistic is

$$z = \frac{\frac{18}{36} - \frac{26}{36}}{0.1119} = -1.99$$

The P-value is given by $2P\left(Z > |-1.99|\right) = 2P(Z < -1.99) = 2(0.0233) = 0.0466$. At 5% level of significance we would reject the null hypothesis in favor of the alternative. That is we accept that there is a difference in the mortality rate between the two surfaces.

# Some items of interest

## How do I remember which tail to look for the P-value?

An easy rule of thumb is that if I rearrange the alternative hypothesis so that the proportions are on one side of the inequality (and are in the same order as in the numerator of your test statistic) and 0 is on other. If it is greater than 0 look at the upper tail if it is less than 0 look at the lower tail. for example consider the second hypothesis test above rearranged

$$H_0 : p_1 - p_2 \leq 0 \text{ vs } H_A : p_1 - p_2 > 0 \text{ P-value } P\left(Z \geq z\right)$$

## How do confidence intervals and hypothesis tests relate to each other?

Confidence levels and two-sided hypothesis tests are related. In particular a level $\alpha$ two sided hypothesis test rejects the null eg $p_1 - p_2 = 0$ when 0 falls outside the level $1 - \alpha$ confidence interval for $p_1 - p_2$. We saw an example of this in question 3 above.