Stat 20 Discussion for section #3

Sept 8, 2003

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Question 1

We play a Wheel of Fortune game. It costs $1 to play the game. There are 6 slots of unequal probability:

Slot	Probability	Pay off	Net
1	0.6	0	-1
2	0.2	1	0
3	0.12	2	1
4	0.05	3	2
5	0.02	10	9
6	0.01	20	19

Let W_1 be amount of net win (pay off -$1).
Let W_2 be amount of net win or 2nd.
Let $W = W_1 + W_2$

What is
(a) $P(W_1 = W_2)$?
(b) $P(W_1 > W_2)$?
(c) $E(W_1)$ $Var(W_1)$?
(d) $E(W_1)$ $Var(W)$?

Answer:

(a) $P(W_1 = W_2) = P(W_1 = W_2 = -1) + \ldots + P(W_1 = W_2 = 19)$
$= 0.6^2 + 0.2^2 + 0.12^2 + 0.05^2 + 0.02^2 + 0.01^2$
$= 0.36 + 0.04 + 0.144 + 0.0025 + 0.0004 + 0.0001$
$= 0.547$

(b) $P(W_1 > W_2) = P(W_2 > W_1) = \dfrac{1 - P(W_1 - W_2)}{2}$

$= \dfrac{1 - 0.547}{2} = \dfrac{0.453}{2}$

$= 0.2265$

(c) $E(W_1) = -1(0.6) + 0(0.2) + 1(0.12) + 2(0.05) + 9(0.02) + 19(0.01)$
$= -0.06 + 0.12 + 0.1 + 0.18 + 0.19$
$= -0.01$

$Var(W_1) = E(W_1)^2 - (E(W_1))^2$

$E(W_1)^2 = (-1)^2 (0.6) + 0^2 (0.2) + 1(0.12) + 4(0.05) + 81 (0.02) + 361 (0.01)$
$= 0.6 + 0 + 0.12 + 0.2 + 1.62 + 3.61$
$= 6.15$

$Var(W_1) = 6.15 - (-0.01)^2 = 6.1499$

(d) $E(W) = E(W_1) + E(W_2) = -0.01 - 0.01 = -0.02$
$Var(W) = Var(W_1 W_2) = Var(W_1) + Var(W_2) = 6.1499 + 6.1499 = 12.2998$

Question 2

Let Z_1, Z_2, and Z_3 be independent random variables, each with mean 0, variance 1.
Let $X = Z + 3Z_1 + 4Z_2 + Z_3$
$Y = 9 - 3Z_1 + Z_2 + 4Z_3$

(a) What is the mean and variance of X?
(b) What is the mean and variance of Y?
(c) What is the mean and variance of 2X + Y?

Answer: Discussed in class. Note that for (c) X and Y are not independent.

Question 3

X, Y, and Z are independent random variables with SD O. Let a, b, and c be constants such that $a^2 + b^2 + c^2 > 0$.

What is the variance of $\dfrac{ax - by - cz}{a^2 + b^2 + c^2}$?

Answer: Discussed in class.

Question 4

We have two scales for measuring weights in a chemistry lab. Repeated weighings of an item on the same scale produces varying measurements.

Scale 1: mean 2.000g SD 0.002

Scale 2: mean 2.001g SD 0.001

Let X be the measurement on scale 1. Let Y be the measurement on scale 2. The readings are independent between scales. What is E(Y – X)? Var (Y – X)?

Let Z= (X + Y)/2. What is E(Z), Var (Z)?
Is the average (Z) more or less variable than Y (the less variable value)?

Answer:

E(Y-X) = 2.001 – 2.000 = 0.001
Var(Y-X) = Var(Y) + Var(X) (independent)
= (0.002)^2 + (0.001)^2
= 0.000005

E(Z) = E((X + Y)/2) = ½ (E(X)+ E(Y)) = 2.0005
Var(Z) = Var(1/2(X+Y)) = ¼ (Var(X) + Var(Y)) = ¼ (0.000005) = 0.00000125

The average Z is less variable than either variable X or Y.

Question 5

Consider two 1 year investments (each of $1000). For the 1st investment we get back $1000 with p = 0.6, $2000 with p = 0.3, and $0 with p = 0.1. For the 2nd investment, $1100 with p = 0.6, $1300 with p = 0.3, and $500 with p = 0.1.

(a) For which investment do we expect to get back more?
(b) Which investment gives us the most variable return?

Answer:

(a) For the 1st investment. The expected value is 0.1* (0) + 0.6*1000 + 0.3*(2000) = $1200 and for the 2nd investment. The expected value is 0.1 * (500) + 0.6* (1100) + 0.3* (1300) = $1100. So the first investment will give us back more (on average).
(b) For the 1st investment the variance is 0.1*(0)^2 + (0.6)*(1000)^2 + 0.3*(2000)^2 = 1,800,000. For the 2nd investment the variance is 0.1*(500)^2 + (0.6)*(1100)^2 + (0.3)*(1300)^2 = 1, 258,000. So the first investment is more variable.