Stat 20: Discussion at section #4

B. M. Bolstad, bolstad@stat.berkeley.edu

Sept 10, 2003

Question 1

A factory employs several thousand workers of whom 30% are Hispanic. If the 15 members of the union executive committee are chosen from all the workers at random.

(a) What is the probability that exactly 3 members are Hispanic?

(b) What is the probability of less than 3 Hispanics being members of the committee?

(c) What is the probability of 3 or more members of the committee being Hispanic?

(d) What is the mean and variance of the number of Hispanics selected for the committee?

Answer: Let Y = number of Hispanics on committee. Then Y is binomial with $n = 15$ and $p = 0.3$

(a) $P(Y = 3) = \binom{15}{3} 0.3^3(1{-}0.3)^{15-3} = \frac{15!}{12!3!}0.3^3(0.7)^{12} = \frac{15(14)(13)}{3(2)(1)}0.3^3(0.7)^{12} = 0.170$ (3dp)

(b)

$$
\begin{aligned}
P(Y < 3) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\
&= \binom{15}{0} 0.3^0(1 - 0.3)^{15} + \binom{15}{2} 0.3^1(1 - 0.3)^{15-1} + \binom{15}{2} 0.3^2(1 - 0.3)^{15-2} \\
&= (0.7)^{15} + 13(0.3)(0.7)^{14} + \frac{15(14)}{2}0.3^2(0.7)^{13} \\
&= 0.127 \text{ (3dp)}
\end{aligned}
$$

(c) $P(Y \geq 3) = 1 - P(Y < 3) = 1 - 0.127 = 0.873$ (3dp)

(d) $E(Y) = np = 15(.3) = 4.5$ and $Var(Y) = np(1 - p) = 15(0.3)(0.7) = 3.15$

Question 2

Suppose we toss a fair coin 10 times and roll a fair six sided dice 6 times then what is the probability of getting 7 heads and the dice showing a 1 or 6 on three of the rolls?

Answer: Let X = number of heads in 10 tosses. So X is distributed Binomial $n_1 = 10$, $p_1 = 0.5$. Let Y = number of times 1 or 6 shows in 6 rolls. So Y is distributed Binomial $n_1 = 6$, $p_1 = 1/3$.

Then $P(X = 7) = \binom{10}{7} 0.5^7(1 - 0.5)^3 = 0.117(3\text{ dp})$ and $P(Y = 3) = \binom{6}{3}(1/3)^3(1 - 1/3)^3 = 0.219(3\text{ dp})$.

Since X and Y are independent. $P(X = 7 \cap Y = 3) = P(X = 7)P(Y = 3) = (0.117)(0.219) = 0.026(3\text{ dp})$.

Question 3

Suppose we toss a fair penny 4 times and a bent quarter 6 times (P(H) = 0.75). What is the expected number of heads? What is the variance?

Answer: Let

X = Number of heads on penny. X is binomial $n_1 = 4$, $P_1 = 0.5$

Y = Number of heads on quarter. Y is binomial $n_1 = 6$, $P_1 = 0.75$

$T = X + Y$ be the total number of heads.

Then $E(X) = 4(.5) = 2$ and $E(Y) = 6(.75) = 4.5$. Similarly $Var(X) = 4(.5)(.5) = 1$ and $Var(Y) = 6(.75)(.25) = 1.125$.

Since $T = X+Y$. It follows that $E(T) = E(X+Y) = E(X)+E(Y) = 2+4.5 = 6.5$. The results from tossing each of the coins are independent therefore $Var(T) = Var(X + Y) = Var(X) + Var(Y) = 1 + 1.125 = 2.125$.

Question 4

Returning to our coin tossing game of previous sections. Remember that this is a game of 4 tosses where we win \$1 for each heads but lose \$1 for each tails.

(a) What is the mean number of heads? How about the variance?

(b) What is our expected winnings? How about the variance?

(c) How about with an unfair coin (P(H) = 0.25). What are (a) and (b) in this case?

(d) How about with an unfair coin and we play for 20 tosses?

Answer: Let X = number of heads in game. So X is Binomial with p = 0.5, n =4.

(a) $E(X) = 4(.5) = 2$ and $Var(X) = 4(.5)(.5) = 1$.

Let W be winnings in \$ from playing games. Since we win \$1 from each H and lose \$1 from each tail (of which there will be $n-X$) our total winnings is $W=(1)X+(-1)(n-X)=2X-n$.

(b) $E(W)=E(2X-n)=2E(X)-n=2(2)-4=0$ and $Var(W)=Var(2X-n)=4Var(X)=4$.

(c) Now $p=0.25$ and so $E(X)=4(.25)=1$ and $Var(X)=4(.25)(.75)=3/4$. So, $E(W)=E(2X-n)=2E(X)-n=2(1)-4=-2$ and $Var(W)=Var(2X-n)=4Var(X)=3$.

(d) Now $p=0.25$ and $n=20$ Thus, $E(X)=20(.25)=5$ and $Var(X)=20(.25)(.75)=3.75$. So, $E(W)=E(2X-n)=2E(X)-n=2(5)-20=-10$ and $Var(W)=Var(2X-n)=4Var(X)=15$.

Miscellaneous

Remember the following for the Binomial distribution.

1. If Y is distributed Binomial with parameters n,p write $Y\sim \text{Bin}(n,p)$.
2. Probability distribution function: $P(Y=k)=\binom{n}{p}p^k(1-p)^{n-k}$
3. Expected Value: $E\left(Y\right)=np$
4. Variance: $Var(Y)=np(1-p)$

Note that $\binom{n}{p}=\frac{n!}{k!(n-k)!}$. Where $n!=n(n-1)(n-2)\ldots 1$. Also remember that $0!=1$.