Stat 20: Discussion at section #5

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Sept 15, 2003

Question 1

If X is normally distributed with mean 30 and standard deviation 10 then what are:

(a) $P(X < 15)$

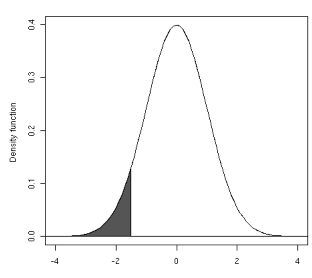
(b) $P(X > 33)$

(c) $P(21 < X < 39)$

Answer:

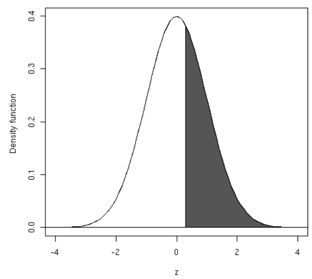
(a)

$$\begin{aligned} P(X < 15) &= P\left(\frac{X-30}{10} < \frac{15-30}{10}\right) \\ &= P(Z < -1.5) \\ &= 0.0668 \end{aligned}$$



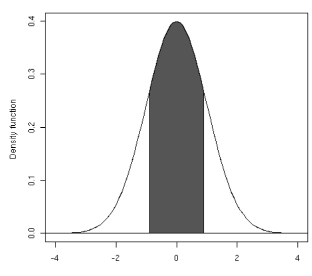
(b)

$$\begin{aligned} P(X > 33) &= P\left(\frac{X-30}{10} > \frac{33-30}{10}\right) \\ &= P(Z > 0.3) \\ &= 1 - P(Z < 0.3) \\ &= 1 - 0.6179 \\ &= 0.3821 \end{aligned}$$



(c)

$$\begin{aligned}
P(21 < X < 39) &= P\left(\frac{21-30}{10} < \frac{X-30}{10} < \frac{39-30}{10}\right) \\
&= P(-0.9 < Z < 0.9) \\
&= P(Z < 0.9) - P(Z < -0.9) \\
&= 0.8159 - 0.1841 \\
&= 0.6318
\end{aligned}$$



Question 2

Let X be normally distributed with mean 5 and variance 4 and Y be normally distributed with mean -1 and variance 16. X and Y are independent. What is $P(Y > X)$? What is $P(Y + 6 > X)$?

Answer: Since X and Y are independent and X and Y are both normally distributed the following apply

1. $E[Y - X] = E[Y] - E[X] = -1 - 5 = -6$ (Note that this applies irrespective of independence or normality.)

2. $Var(Y - X) = Var(Y) + Var(-X) = Var(Y) + (-1)^2 Var(X) = 16 + 4 = 20$ (Note that this depends on independence).

3. $Y - X$ will be normally distributed with mean -6 and sd $\sqrt{20}$. (note this depends on both normality and independence)

So returning to our problem

$$\begin{aligned}
P(Y > X) &= P(Y - X > 0) \\
&= P\left(\frac{Y - X - -6}{\sqrt{20}} > \frac{0 - -6}{\sqrt{20}}\right) \\
&= P\left(Z > \frac{6}{\sqrt{20}}\right) \\
&= P\left(Z > 1.34\right) \\
&= 1 - P(Z < 1.34) \\
&= 1 - .9099 = 0.0901
\end{aligned}$$

And similarly

$$\begin{aligned}
P(Y + 6 > X) &= P(Y - X > -6) \\
&= P\left(\frac{Y - X - -6}{\sqrt{20}} > \frac{6 - -6}{\sqrt{20}}\right) \\
&= P\left(Z > 0\right) \\
&= 1 - P(Z < 0) \\
&= 1 - .5 \\
&= 0.5
\end{aligned}$$

Question 3

Let Y be be normally distributed with mean 1 and sd 3. What is $P(20 \leq 6 + 3Y \leq 28)$?

Answer:

$$\begin{aligned}
P(20 \leq 6 + 3Y \leq 28) &= P(14 \leq 3Y \leq 22) \\
&= P\left(\frac{14}{3} \leq Y \leq \frac{22}{3}\right) \\
&= P\left(\frac{\frac{14}{3} - 1}{3} \leq Z \leq \frac{\frac{22}{3} - 1}{3}\right) \\
&= P(1.22 \leq Z \leq 2.11) \\
&= P(Z < 2.11) - P(Z < 1.22) \\
&= .9826 - .8888 \\
&= 0.0938
\end{aligned}$$

Question 4

We make a bet on a game. You bet me that your score will be at least twice my score. Suppose that scores are normally distibuted and that our scores are independent. My scores

X are normally distributed with mean 100 and sd 10. Your scores Y are normally distributed with mean 180 and sd 20. What is the probability that you win the bet?

Answer: To win the bet you require that $Y > 2X$ so we are interested in $P(Y > 2X)$.

Since X and Y are independent and X and Y are both normally distributed the following apply

1. $E[Y - 2X] = E[Y] - 2E[X] = 180 - 200 = -20$ (Note that this applies irrespective of independence or normality.)

2. $Var(Y - 2X) = Var(Y) + Var(-2X) = Var(Y) + (-2)^2 Var(X) = 400 + 4(100) = 800$ (Note that this depends on independence).

3. $Y - 2X$ will be normally distributed with mean -20 and sd $\sqrt{800}$. (note this depends on both normality and independence)

Returning to the problem

$$\begin{aligned}
P(Y > 2X) &= P(Y - 2X > 0) \\
&= P\left(\frac{Y - 2X - -20}{\sqrt{800}} > \frac{0 - -20}{\sqrt{800}}\right) \\
&= P(Z > 0.71) \\
&= 1 - P(Z < 0.71) \\
&= 1 - 0.7611 \\
&= 0.2389
\end{aligned}$$