Stat 215b (Spring 2004): Lab 4

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Note that you have three weeks to do this lab because of the Spring Break week. There will be no extensions for this lab.

Introduction

In this lab we will explore two topics: non-linear regression and the bootstrap.

Part I - Non-linear regression

A sample of blood is taken, labeled radioactively and re-injected. Then, further samples, all of the same size, are taken at specific times after the re-injection, and a radioactive count is taken of each. Such procedures are used to assess the relative lifetimes of blood cells treated differently. The data file `radiation.dat` on the web site includes times and counts for one case.

We want to consider the nonlinear model

$$\mathrm{E}[\text{count}] = \alpha + \beta \exp(-\gamma t)$$

where $t \geq 0$ refers to time, and α, β and γ are unknown parameters. The parameter α is the background radiation level.

1. Plot the data and comment on its form.

2. Estimate the parameters using nonlinear regression. Try several sets of initial values. Does the program converge to the same place each time? Can you make it diverge?

3. Assume that the model error is i.i.d. normal. Use the estimated information matrix to estimate the variance/covariance matrix of the estimator.

4. Is there sufficient evidence to suggest that background radiation is present (i.e. that $\alpha > 0$)? Give a 95% confidence interval for α.

5. Assess the fit of the model using residual plots, etc.

6. The expected radioactive count at time 0 is $\alpha + \beta$. In the limit, as $t \to \infty$, the expected count is α. Estimate the time, $T_{1/2}$, at which the expected count is $\alpha + \beta/2$. Give an estimate of the standard error. Suppose that for normal blood, $T_{1/2} = 40$. For the current data, is there sufficient evidence to suggest that $T_{1/2} \neq 40$?

7. Counts of radioactive particles are usually modeled as Poisson. Is that indicated by this data?

Part II - Bootstrapping

1. Assess the appropriateness of the uncertainties used in Part I. To do this assume that the normal error model is true and that the underlying parameter values are exactly the values of your estimates. Simulate data from this model and re-estimate the parameters. Repeat this many times to get samples of the estimates (for the parameters). This process is called the parametric bootstrap. What is the variability of the estimates in this case.

2. Suppose instead of assuming a parametric model to generate the errors, re-sample from the observed residuals. Repeat the process. This is called the non-parametric bootstrap. What are the bootstrap estimates of the variability in this case?

3. **Optional** You might want to consider what happens if you use a Poisson model for the truth.