Stat 215b (Spring 2004): The Delta Method

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Theory

Suppose that T_n is an estimator of θ. If

$$\sqrt{n}\left[T_n - \theta\right] \stackrel{D}{\rightarrow} N\left(0, \tau^2\right)$$

then

$$\sqrt{n}\left[h\left(T_n\right) - h\left(\theta\right)\right] \stackrel{D}{\rightarrow} N\left(0, \tau^2\left[h'\left(\theta\right)\right]^2\right)$$

provided $h'\left(\theta\right)$ exists and is not equal to zero.

Example 1

Suppose we consider the sample extimator $\bar{x}$ of μ for normal data with known standard deviation σ. Suppose we are instead interested in μ^2. What is the standard error of $\bar{x}^2$?.

$$\frac{df}{d\mu} = 2\mu$$

so

$$\mathrm{var}\left(\mu^2\right) = 4\mu^2\frac{\sigma^2}{n}$$

which we would estimate with

$$4\bar{x}^2\frac{\sigma^2}{n}$$

Some simulation code to check example 1

```
nreps <- 1000
nsamps <- 100


x.sim <- matrix(0,nreps,nsamps)
for (i in 1:nreps){
  x.sim[i,] <- rnorm(nsamps,mean=10,sd=1)
```

```
}

hist(apply(x.sim,1,mean))
var(apply(x.sim,1,mean))

hist(apply(x.sim,1,mean)^2)
var(apply(x.sim,1,mean)^2)    #should be close to 4
```

Example 2

Suppose we consider the sample extimator $\bar{x}$ of μ for normal data with known standard deviation σ. Suppose we are instead interested in $1/\mu^2$. What is the standard error of $\frac{1}{\bar{x}^2}$?.

$$\frac{df}{d\mu} = \frac{-2}{\mu^3}$$

so

$$\mathrm{var}\left(\mu^2\right) = \frac{4}{\mu^6}\frac{\sigma^2}{n}$$

which we would estimate with

$$\frac{4}{\bar{x}^6}\frac{\sigma^2}{n}$$

Some simulation code to check example 1

```
nreps <- 1000
nsamps <- 100


x.sim <- matrix(0,nreps,nsamps)
for (i in 1:nreps){
  x.sim[i,] <- rnorm(nsamps,mean=10,sd=1)
}

hist(apply(x.sim,1,mean))
var(apply(x.sim,1,mean))

hist(1/apply(x.sim,1,mean)^2)
var(1/apply(x.sim,1,mean)^2)    #should be close to 4*10^-8
```